Compensation of Self Phase Modulation by Anomalous Dispersion in Nonlinear Optical Communication Systems

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Abstract

In nonlinear optical systems, self phase modulation gives positive frequency chirp which increases pulse broadening. Anomalous group velocity dispersion in fibers gives negative frequency chirp. The positive frequency chirp due to self phase modulation is compensated by negative frequency chirp due to group velocity dispersion. In order to minimize the effect of self phase modulation in optical systems the proper group velocity dispersion is required. In this paper the effect of positive and negative group velocity dispersion is shown on self phase modulation. The simulated results show that the interaction of anomalous group velocity dispersion with self phase modulation gives improved system performance. Hence in nonlinear optical system the self phase modulation can be compensated by anomalous dispersion.

Keywords: Nonlinear optics, self phase modulation, group velocity dispersion

1. Introduction

Recent progress in the improvement of optical fibers has transformed the field of telecommunication of which fiber-based networks form an integral part. Guidance of light through optical fiber takes place by the phenomenon of total internal reflection. Though there are losses to be incurred during the transmission, they have been reduced significantly by using silica fibers (less than 20dB/km) and advances in fabrication technologies (Kao, et al, 1986).

Another very significant development involves use of optical nonlinear effects in optical fibers to compensate for any type of distortion of signal due to dispersion in the fiber. The field of nonlinear optics continued to grow up where optical fibers were used to make amplifiers and lasers. The advent of fiber amplifiers increased fiber transmission capability. Although GaAlAs-based solid state optical amplifiers were the first of their kind (Yamamoto, et al, 1989), most successfully and widely used devices are erbium-doped fiber amplifiers (EDFA), working at 1550 nm (Desurvire, 1994). Most currently installed systems are based on communication at a 1300 nm optical window of transmission. This was due to the phenomenon that around an operating wavelength of 1300 nm the optical pulses propagate through a conventional single-mode fiber with almost no pulse broadening. But as silica fibers have lowest attenuation in the 1550 nm wavelength band, special fibers known as dispersion-shifted fibers have been developed which provide negligible dispersion in the 1550 nm band.

Nonlinear effects in optical fibers can manifest qualitatively different behavior depending in the sign of group velocity dispersion (GVD) parameter (Agrawal, 2007). Though combination of both GVD and self phase modulation (SPM) generates soliton pulses, individual effects have also found major applications in pulse compression, chirped-pulse amplification, passive mode-locking and in fast optical switching. This paper deals with effects of SPM and GVD on a system, with SPM compensated by GVD. It is organized as follows: Section II deals with the important effects in an optical fiber such as basic nonlinearity, GVD and SPM along with their chirping effects. Section III describes the experiment and simulation results. Conclusions are presented in Section IV.

2. Nonlinear Effects in Optical Fiber Systems

For most of the analysis regarding propagation, the optical fiber is considered to be linear medium. The intensities associated with the propagation of optical pulse are assumed to be so small that no significant effect can be considered on the propagation characteristics of the waveguide. But practically, all media exhibit nonlinear effects. In case of silica fibers, one of the visual effects of the nonlinearity is intensity-
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... dependent refractive index. This intensity-dependent refractive index leads to the phenomenon of SPM. The primary effect of SPM is to broaden the spectrum while keeping temporal distribution unaltered. Dependency of group velocity on frequency gives rise to GVD. It leads to broadening of pulse in the time domain keeping the spectral content the same.

2.1 Basic Nonlinearity

For optical fibers, the response becomes nonlinear for dense electromagnetic fields due to aharmonic motion of bound electrons under the influence of applied field. This nonlinearity can degrade the transmission by involving two major phenomena namely group velocity dispersion (GVD) and self phase modulation (SPM). The following are thus important challenges while designing such networks:

1) Transmission of different wavelengths at highest possible bit rate
2) Transmission over long distances with less optical amplifiers

Nonlinearity originates due to two processes: nonlinear inelastic scattering process and intensity dependent variations. Nonlinear inelastic scattering processes are due to interactions between the optical signals and molecular or acoustic vibrations in a fiber. It includes Stimulated Raman Scattering (SRS) and Stimulated Brillouin Scattering (SBS). For intensity dependent variations which arises due to nonlinear refractive index self phase modulation (SPM), cross phase modulation (XPM) and four wave mixing (FWM). When any of these nonlinear effects contribute to signal impairment, an additional amount of power will be needed at the receiver to maintain the same bit error rate (BER) as in their absence (Keiser, 1999). Hence it demands more power. In case of processes like SBS, SRS and FWM, this additional power is depleted from another channels, thereby producing crosstalk. It degrades carrier-to-noise ratio. In case of SPM and XPM, it affects the phase of signals, thereby producing chirp.

For higher order nonlinear effects, if peak power of incident pulse is above threshold value then SRS and SBS can transfer energy to a new pulse at different wavelength, which may propagate in same or opposite direction (Agrawal, 2007). This pulse propagation depends upon nonlinear parameter $\gamma$ which is defined as

$$\gamma(\omega_0) = \frac{\eta_2(\omega_0) \omega_0}{cA_{eff}}$$

(1)

where $\eta_2(\omega_0)$ is the nonlinear refractive index, $\omega_0$ is center frequency for pulse spectrum Generally, $\omega_0 \approx 10^{15}$ s$^{-1}$.

When an optical pulse propagates through a linear dispersive medium it undergoes temporal broadening as well as chirping. When the pulse is propagating through a nonlinear nondispersive medium there is no temporal broadening but only chirping. Figure 1 shows propagation of pulse through both the mediums and their effects on pulse.

Fig. 1 Effects of linear dispersive and nonlinear nondispersive medium on propagation of pulse (Ghatak, et al, 2002).

2.2 Self Phase Modulation

The refractive index $n$ of many optical materials has dependency on optical intensity $I$ which is given as

$$n = n_0 + n_2 \frac{P}{A_{eff}}$$

(2)

where $n_0$ is the ordinary refractive index, $n_2$ is the nonlinear index coefficient

This nonlinearity in refractive index produces carrier-induced phase modulation of propagating signal. It converts optical power fluctuations to spurious phase fluctuations.

Edges of the pulse represent time-varying intensity which rises rapidly from zero to maximum value and then returns to zero. If it is a medium where refractive index depends on intensity, then it will produce time-varying refractive index. The phase fluctuations being intensity-dependent, different parts of pulse have different phase shifts.

Fig. 2 Temporal variations of SPM-induced phase shift (Agrawal, 2007)
Normalized amplitude of pulse is given as
\[ U(L, T) = U(L, T)\exp[i\phi_{NL}(L, T)] \]  \hspace{1cm} (3)

And nonlinear phase shift is given as
\[ \phi_{NL}(L, T) = \frac{E_{NL}L_{eff}}{L_{NL}} \]  \hspace{1cm} (4)

Thus for maximum phase shift \( \phi_{max} \) occurring at \( T = 0 \), amplitude is given as
\[ |U(0, 0)| = 1 \]  \hspace{1cm} (5)

For which phase is given by
\[ \phi_{max} = \frac{L_{eff}}{L_{NL}} = \gamma P_0 L_{eff} \]  \hspace{1cm} (6)

This equation shows that SPM gives rise to intensity-dependent phase shift.

SPM alone broadens the signal spectrum, but does not affect the intensity profile of the signal. The spectral broadening effect can be understood from the fact that the time-dependent phase variation causes instantaneous frequency deviation \( \partial \omega(t) \) which is given by
\[ \partial \omega(t) = \frac{\partial^2 \phi}{\partial t^2} = -\frac{2m_nz_{eff}}{\lambda} \frac{\partial I(t)}{\partial t} \]  \hspace{1cm} (7)

Where \( z_{eff} \) is the effective transmission distance taking into account of the fiber attenuation, \( z \) is propagating distance and \( I(t) \) is optical intensity in the units of \( \text{W/m}^2 \). This equation suggests that the magnitude of instantaneous frequency deviation increases with distance and the intensity variation of the signal (Wongpaibool, 2003).

When the SPM is sufficiently strong, a pulse can be compressed during initial propagation. In addition, under ideal conditions the pulse shape can be continually preserved during propagation due to the balancing of the effects of dispersion and SPM in the anomalous dispersion regime. This type of pulse is called a soliton (Hasegawa, et al., 1973), (Hasegawa, 2000), (Nakazawa, 2000). However, the effect of SPM decreases with transmission distance due to fiber attenuation, preventing soliton formation (Jacobs, et al., 2001).

For a Gaussian input optical pulse given by
\[ E(z = 0, t) = E_0 e^{-t^2 / 2 \tau^2} e^{i0t} \]  \hspace{1cm} (8)

First term on the LHS represents wave travelling through dispersive medium. Second term is

\[ E(z, t) = \frac{E_0}{(1 + \sigma^2)^{1/4}} \exp \left[-\frac{(t - z)^2}{\tau^2(z)}\right] \]  \hspace{1cm} (9)

From the phase term in above equation, it follows that the oscillations within output pulse are not periodic. From the same equation, instantaneous frequency within the pulse envelope is obtained as
\[ \omega(t) = \frac{\partial \phi}{\partial t} = \omega_0 + 2k \left(t - \frac{z}{v_g}\right) \]  \hspace{1cm} (10)

Thus the instantaneous frequency within the pulse envelope changes with time. Such a pulse is termed as chirped pulse.

![Fig.3 Propagation of pulse broadens it in time domain and also gets chirped (Ghatak, et al, 2002).](image)
proportional to $\alpha$ and is a dispersion term and the last term represents nonlinearity.

In the absence of nonlinearity, this equation will only represent GVD and chirping due to GVD. Thus modifying the equation for dispersive effects

$$-i\left(\frac{\partial f(z,T)}{\partial z}\right) - \frac{1}{2} \alpha \frac{\partial^2 f(z,T)}{\partial T^2} = 0$$  \hspace{1cm} (12)

Using the method of separation of variables, general solution of above equation is given as

$$f(z,T) = \int A(\Omega)e^{i(\Omega T - \frac{1}{2} \gamma \mu_z^2)} d\Omega$$  \hspace{1cm} (13)

$A(\Omega)$ represents frequency spectrum of the input pulse. Due to dispersion, the pulse experiences chirping.

In the linear approximation, the envelope propagates with the group velocity $v_g$ remaining unchanged in shape. But in the quadratic approximation it spreads and reduces in amplitude with distance $z$ and it chirps as shown in figure 4.

![Fig. 4 Pulse spreading and chirping due to GVD](image)

### 2.4 Different Propagation Routines

Non-linear effects in an optical fiber are governed by non-linear length $L_{NL}$ and dispersive effects are governed by dispersion length $L_D$, which is given as follows:

$$L_D = \frac{T_o^2}{\beta_2} \text{ and } L_{NL} = \frac{1}{\gamma P_0}$$  \hspace{1cm} (14)

where $T_o$ is the initial width of incident pulse $P_o$ is the peak power of incident pulse $\beta_2$ is the GVD parameter $\gamma$ is the nonlinear parameter $L_D$ and $L_{NL}$ provide the length scales over which dispersive or nonlinear effects become important for pulse evolution. Depending on the relative magnitudes of $L_D$, $L_{NL}$ and $L_{NL}$, the propagation behavior can be classified into four categories:

1. When $L << L_{NL}$ and $L << L_D$

Neither dispersion nor nonlinear effects play a significant role during pulse propagation. Pulse maintains its shape during propagation and has a smooth temporal profile. Fiber acts as mere transporter of optical pulse except for reducing the pulse energy.

2. When $L << L_{NL}$ and $L \sim L_D$

As fiber length becomes comparable with nonlinear length, dispersive effects play a relatively minor role. Mathematically,

$$\frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_o^2}{|\beta_2|} << 1$$  \hspace{1cm} (15)

3. When $L \sim L_{NL}$ and $L << L_D$

As fiber length becomes comparable with nonlinear length, nonlinear effects take place. Dispersive effects play relatively minor role. Mathematically,

$$\frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_o^2}{|\beta_2|} >> 1$$  \hspace{1cm} (16)

4. When $L > L_{NL}$ and $L > L_D$

For this case, dispersion and nonlinearity act together as pulse propagates along the fiber.

### 3. Simulation

An optical system was considered for the experiment which was simulated using OptiSim. The block diagram of the system is as shown in figure 5.

It consisted of a data source which was stochastic in nature. It was modulated by an NRZ modulator driver so that it could be propagated through a modulator. Modulator considered for experimental purpose was sine square modulator. It was driven by an optical source, continuous wave laser.

This modulated signal was then propagated through an optical fiber of certain length. Optical filter was used to selectively receive particular wavelengths of light. To amplify the optical signal directly without converting it to electrical signal, optical amplifier having 35dB of maximum small signal gain was used. It was later converted into equivalent electrical signal by means of a photodiode. Eye diagram was observed for the purpose of signal distortion analysis.

For fixed wavelength and fixed fiber length, eye diagram and related factors were observed for three different values of GVD parameter. Table 1 gives values of each factor for a given GVD parameter.

### Table 1 Various factor for fixed wavelength ($\lambda = 1550\text{nm}$) and fixed fiber length ($L = 100\text{km}$) system for three different GVD parameters

<table>
<thead>
<tr>
<th>GVD parameter (ps/nm.km)</th>
<th>Q factor (dB)</th>
<th>BER</th>
<th>Jitter (ns)</th>
<th>Eye Closure (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>6.02</td>
<td>0.0227501</td>
<td>0.0260187</td>
<td>22.007920</td>
</tr>
<tr>
<td>0</td>
<td>6.02</td>
<td>0.0227501</td>
<td>0.0236062</td>
<td>20.455021</td>
</tr>
<tr>
<td>-10</td>
<td>40</td>
<td>1e-40</td>
<td>0.0187679</td>
<td>0.029480</td>
</tr>
</tbody>
</table>
For all the three values of GVD parameter in table 1, eye diagrams are shown in figures 6 (a), (b) and (c).

**Conclusions**

In this paper the effects of GVD on SPM are shown. The system performance is analyzed for different GVD parameters. It was found that the system performance degrades with increasing GVD parameter. It is also found that GVD with negative dispersion coefficient compensates SPM in nonlinear optical system. Hence this is the simplest technique to enhance the system performance in presence of SPM.

Future work can be done in order to generate and analyze solitons in optical fiber system which is the best possible application of combination of SPM and GVD both.

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Fig. 5 Block diagram of simulated system

Fig. 6(a) Eye diagram for L= 100 km and GVD parameter= +10 ps/nm.km

Fig. 6(b) Eye diagram for L=100 km and GVD parameter= 0 ps/nm.km

Fig. 6(c) Eye diagram for L= 100 km and GVD parameter= -10 ps/nm.km